- 全卷試題分爲三大項:
  - 壹、是非題: 題號(1) 至 (10), 共10題, 總分20分。
  - 貳、配合題與填充題: 題號(11) 至 (17), 共7題, 總分25分。
  - 參、計算題與證明題: 題號(18) 至 (29), 共12題, 總分80分。

  試題總分共125分, 成績計算方式 = 得分 × 0.8, 再以四捨五入在個位取概數。
- 將答案寫於答案簿, 並標示正確的題號。[壹]和[貳]的答案請依序寫在答案簿之前兩頁。[參]的答案從答案簿第三頁寫起, 題序不拘。
- 不准使用計算器。

壹、是非題

- 本大題每題2分, 共20分。
- 答題請用 ○ 表示「是」、× 表示「非」。任何解釋均不計分。

(1) If $f(x)$ is a continuous function defined on the interval (a, b), the range of f must be an open interval.

(2) The equation $x^7 - 8x + 5 = 0$ has a root in the interval $(0, 2)$.

(3) If f is differentiable, then $\frac{d}{dx} f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$.

(4) There is a function f such that $f(x) > 0$, $f'(x) < 0$ and $f''(x) < 0$ for all $x \in \mathbb{R}$ (the real number).

(5) $\lim_{a \to 0} \frac{\sin^{-1} a}{a} = 1$.

(6) Let $f(x)$ be a real valued function defined on the real line such that its derivative $f'(x)$ exists everywhere. If $f(x)$ is strictly increasing then $f'(x) > 0$ everywhere .

(7) Let $f(x)$ and $g(x)$ be real valued functions defined on $\mathbb{R}$ such that their derivatives exist everywhere. If $\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = 0$ and $\lim_{x \to 0} \frac{f'(x)}{g'(x)}$ does not exist, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ does not exist, either.

(8) Let $f(x)$ be a real valued function defined on $\mathbb{R}$ such that its derivative $f'(x)$ exists everywhere. If a is the *only* real number such that $f'(a) = 0$ and $f''(a) > 0$, then $f(x)$ achieves its absolute minimum at $x = a$.

(9) If $g(x) \le f(x)$ for $0 \le x < \infty$ and $\int_0^\infty g(x)dx$ diverges, then $\int_0^\infty f(x)dx$ also diverges.

(10) Suppose the continuous function $f(x)$ satisfies the property $f(-x) = -f(x)$ (i.e. an odd function). The integral $\int_{-\infty}^{\infty} f(x)dx = 0$.

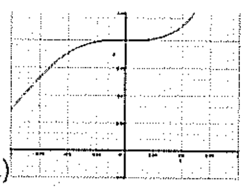
見背面

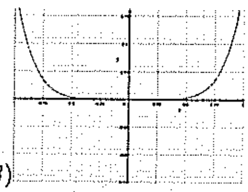
貳、配合題與填充題

- (11)-(14) 為配合題, 每格正確答對者得1分,(11)-(13) 滿分為3分,(14) 為4分。(15)-(17) 為填充題, 每題4分。本大題總分為25分。
- 在答案簿寫下答案, 配合題請照試題畫出簡單表格填答, 填充題請直接寫出答案。任何解釋均不計分。

(11) Mathiching each function: (a) $f(x)$, (b) $e^{f(x)}$, (c) $f(e^x)$ (d) $f(x^2)$ with its graph below.

(a)	(b)	(c)	(d)


(A)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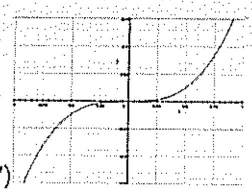
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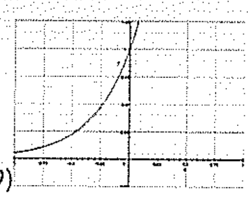
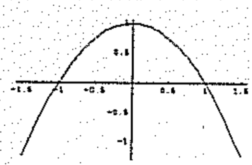
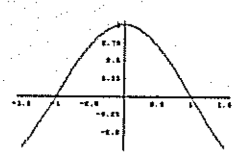

(D)

(12) Match each function: (a) $y = e^{-x^2}$, (b) $y = \cos(\frac{\pi x}{2})$, (c)$y = 1 - x^2$, (d) $y = x^4 - 3x^2 + 1$ with its graph below on the interval [-1.5, 1.5].

(a)	(b)	(c)	(d)


(A)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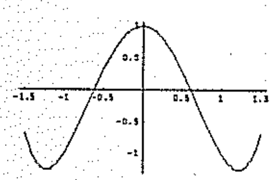
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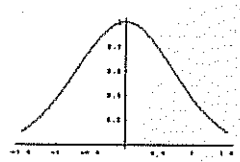
(D)

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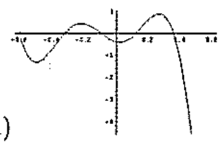
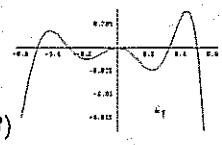
(13) Given a function $y = f(x)$, the figures show the graphs of (a) $f(x)$, (b) $f'(x)$, (c) $f''(x)$, (d) $\int_0^x f(t)dt$ on the interval $[-0.6, 0.6]$. Identify each curve.

(a)	(b)	(c)	(d)


(A)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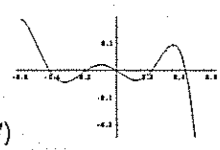
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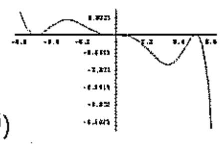
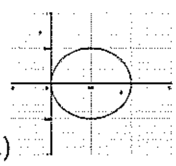
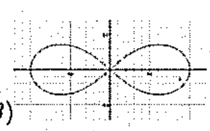

(D)

(14) Matching each polar curves: (a) $r = \cos\theta$, (b) $r = \cos 2\theta$, (c) $r = \cos 3\theta$, (d) $r = \cos 4\theta$ with its graph below.

(a)	(b)	(c)	(d)


(A)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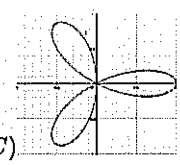
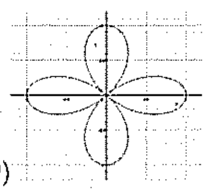
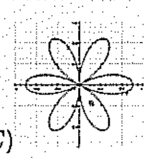
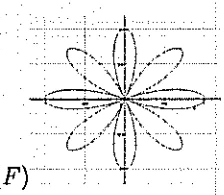

(C)


(D)


(E)


(F)

(15) If the function $y = f(x)$ satisfies $xy + e^y = e$, then $y''(0) =$ ____(15)____

(16) $\int_0^\pi (\sin x + \sqrt[3]{\cos x}\sqrt{2 - \sin x})\, dx =$ ____(16)____.

(17) The volume of the region bounded by the surface $\frac{x^2}{4} + (\sqrt{y^2 + z^2} - 6)^2 = 1$ is equal to ____(17)____.

參、計算題與證明題

- 本大題總分爲80分，每題配分標於題號之後。
- 請完整與清晰的寫下計算或證明過程。

(18) [5pt] Compute $\frac{d^{10}}{dx^{10}}\left(\frac{1+x}{\sqrt{1-x}}\right)$.

(19) [5pt] Find the value of C for which the integral $\int_0^\infty \left(\frac{x}{x^2+1} - \frac{C}{3x+1}\right)dx$ converges. Evaluate the integral for this value of C.

(20) [6pt] Find $\lim\limits_{x\to\infty} \frac{e^x}{((1+\frac{1}{x})^x)^x}$.

(21) [6pt] Let

$$H(x) = \begin{cases} a\sin(2x) + b\cos(3x), & x < 0, \\ m, & x = 0, \\ e^{-\frac{4}{x^2}} + \sqrt[3]{2+x}, & x > 0. \end{cases}$$

Find the constants m, a, b such that $H(x)$ is differentiable everywhere.

(22) [6pt]

(a) compute $\int_0^1 \ln(1+x)dx$.

(b) By above result and using Riemann sum, find the limit

$$\lim_{n\to\infty}\left(\frac{(2n)!}{n!\,n^n}\right)^{\frac{1}{n}}$$

(23) [6pt] Let $p(x) = (x-10)(x-\pi^2)^2(x-\sqrt{97})^3(x-\sqrt[3]{960})^4$ be a polynomial of degree 10. It has 10 real roots, show that the derivative $p'(x) = 0$ has 9 real roots.

(24) [7pt] Compute (i) $\int_0^{\pi/4} \tan^7 x\, dx$; (ii) $\int \frac{1}{1+\sin x}dx$.

(25) [7pt] Find the length of the curve $y = \sqrt{x-1}$ from $x = 1$ to $x = \frac{10}{9}$.

(26) [7pt] Let C be the curve defined by $x(x^2+y^2) = (x^2-y^2)$. (i) Find a parametrization of C by setting $y = tx$; (ii) Compute the area of the region enclosed by the loop of the curve.

(27) [7pt] Consider the sequence given by the recursive relation

$$x_{n+1} = x_n - \frac{\sin x_n}{\cos x_n},\ x_1 = \frac{\pi}{4}.$$

Does this sequence converge? Detail the reason for your answer? If your answer is "yes," what is the limit of x_n as n goes to infinity?

(28) [8pt] Let C be the intersection curve of the surface $y - x^2 = 0$ and the xy-plane $z = 0$ in the space. Let L be the line passing through the point $(1,1,3)$ whose direction is parallel to the vector $(1,2,1)$. Find a point P on C and a point Q on L such that the distance between P and Q is the shortest possible distance between a point on C and a point on L. Explain why your answer gives the shortest possible distance.

(29) [10pt] Sketch the graph of $f(x) = \frac{x}{e^x(x-4)}$. (You must discuss the following: asymptotes; intervals of increase or decrease; local maximum and minimum values; intervals of concavity and the inflection points).

試題隨卷繳回