- 全卷試題分為四大項：**壹、是非題**；**貳、配合題**；**參、填充題**；**肆、計算題與證明題**。試題總分共 125 分，成績計算方式 = 得分 × 0.8，再在個位四捨五入。
- 將答案寫於 試 卷 ，並標示正確的題號。**[壹]**、**[貳]**、**[參]** 的答案請依序寫在 試卷 之前兩頁。**[肆]** 的答案從 試 卷 第三頁寫起，題序不拘。
- 不准使用計算機、手機、平版。

壹、是非題

- 本大題每題 2 分，共 20 分。
- 答題請用 ○ 表示「是」、× 表示「非」。任何解釋均不計分。

(1) There exists a differentiable function on $(0,2)$ such that

$$f'(1) \neq \frac{f(b)-f(a)}{b-a}, \quad \text{for all } 0 < a < 1 < b < 2.$$

(2) If $f(x)$ is a differentiable function, so is $|f(x)| \cdot \sin(f(x))$

(3) Suppose that f is continuous on $[a,b]$ and $c \in (a,b)$ is a point such that $\lim_{x\to c^-} f'(x)$ dosen't exist. Then f is not differentiable at c.

(4) If $\lim_{x\to\infty} \frac{f(x)}{x} = 0$, then $f(x)$ has a horizontal asymptote.

(5) Let $f(x) = \frac{P(x)}{Q(x)}$ be a rational function where $P(x)$ and $Q(x)$ are polynomials. The graph $y = f(x)$ has a slant asymptote if and only if $\lim_{x\to\infty} \frac{f(x)}{x}$ converges to a nonzero number.

(6) Suppose that $f(x)$ is a differentiable function, and on an interval I the graph of f lies above all of its tangents. Then $f'(x)$ is increasing on I.

(7) Suppose the continuous function $f(x)$ satisfies the property $f(-x) = -f(x)$ (i.e. an odd function). The integral $\int_{-\infty}^{\infty} f(x)dx = 0$.

(8) $\int_0^2 \tan^{-1}(\sin x)\, dx \leq \frac{\pi}{2}$.

(9) If f is continuous on $[0,1]$, then $\lim_{n\to\infty} \int_0^1 x^n f(\sqrt[n]{x})dx = 0$.

(10) If f is integrable on $[a,b]$, then $g(x) = \int_x^b f(t)dt$ is differentiable on (a,b) and $g'(x) = -f(x)$ for all $x \in (a,b)$.

接 背 面

題號：2011

貳、配合題

- (11)-(14) 每格正確答對者得 1 分，每題滿分為 3 分。本大題總分為 12 分。
- 請照試題在答案簿上畫出簡單表格填答。任何解釋均不計分。

(11) Match each equation

(a) $f(x) = -|x \sin x|$ (b) $g(x) = \sin\dfrac{1}{x}$ (c) $h(x) = \dfrac{\sin x}{x}$ (d) $k(x) = \dfrac{\sin x}{|x|}$

with its property below.

(a)	(b)	(c)	(d)

(A) It has both the absolute maximal value and absolute minimal value.

(B) It does not have any absolute extreme values.

(C) It has the absolute maximal value, but has no absolute minimal value.

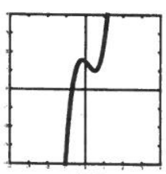
(D) It has the absolute minimal value, but has no absolute maximal value.

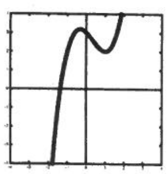
(12) Match each function:

(a) $f(x)$ (b) $\dfrac{f(x)+f(-x)}{2}$ (c) $\dfrac{f(x)-f(-x)}{2}$ (d) $\dfrac{f(2x)}{2}$

with its graph below.

(a)	(b)	(c)	(d)


(A)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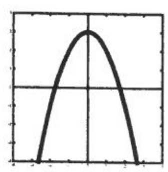
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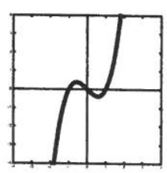

(D)

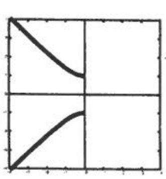
(13) Match each equation

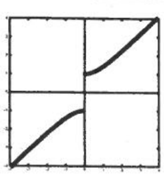
(a) $\begin{cases} x &= -\sqrt{t^2-1} \\ y &= t \end{cases}, |t| \geq 1$ (b) $\begin{cases} x &= \tan t \\ y &= \sec t \end{cases}, \dfrac{\pi}{2} < t < \dfrac{3\pi}{2}$

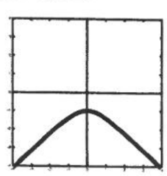
(c) $\begin{cases} x &= \sinh t \\ y &= \cosh t \end{cases}, t \in \mathbb{R}$ (d) $\begin{cases} x &= \dfrac{2t}{1-t^2} \\ y &= \dfrac{1+t^2}{1-t^2} \end{cases}, t \geq 0$

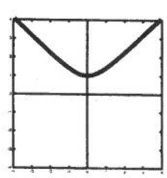
with its graph below.

(a)	(b)	(c)	(d)


(A)


(B)


(C)


(D)

(14) There are four functions defined on $x \in (0, \pi/4)$ and labeled as follow,

$$\text{(a) } \sin x \qquad \text{(b) } \cos x \qquad \text{(c) } 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 \qquad \text{(d) } x - \frac{1}{6}x^3$$

We rearrange the order of these functions to make them satisfy the inequality

$$\text{(A)} > \text{(B)} > \text{(C)} > \text{(D)}$$

Fill in the proper correspondence.

(a)	(b)	(c)	(d)

參、填充題

- (15)-(17) 每題 4 分。本大題總分為 12 分。
- 請直接在答案簿寫下答案。任何解釋均不計分。

(15) Suppose $60 \le \alpha \le 100$. We want to identify a funcion $f(x)$ satisfies the following conditions:

(a) Function $f(x)$ defined over $\mathbb{R}$ is of the form:

$$f(x) = \begin{cases} ax^2 + bx + c, & x \le \alpha \\ x, & \alpha < x \end{cases}$$

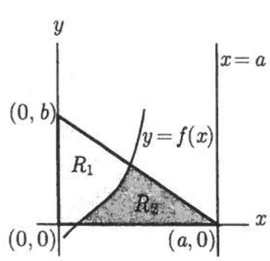
(b) $f(x)$ is continuous.

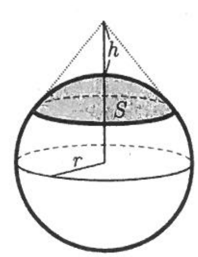
(c) $f'(x)$ is continuous.

(d) The absolute minimum value of $f(x)$ is 60.

Suppose the minimum occur at $x = \beta$, then $\alpha + \beta =$ __________.

(16) Let T be the triangle with vertices $(0,0)$, $(a,0)$ and $(0,b)$ where $a, b > 0$(graphed below left). Suppose that a curve $y = f(x)$ splits T into two regionsR_1 and R_2, with equal area. Let S_1 be the solid obtained by rotating R_1 with respect to the y-axis. Let S_2 be the solid obtained by rotating R_2 with respect to the line $x = a$. The difference of the volumes of S_1 and S_2, $|V(S_1) - V(S_2)|$, is __________.





(17) Let S be the part of a sphere with radius r that an observer at height h above the north pole can see. The area of S is __________.

肆、計算題與證明題

- 本大題總分為 81 分，每題配分標於題號之後。
- 請完整與清晰的寫下計算或證明過程，只寫答案不計分。

(18) [12pt] Find the limits:

(a) $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left[\left(x+\frac{a}{n}\right)^2+\left(x+\frac{2a}{n}\right)^2+\cdots+\left(x+\frac{(n-1)a}{n}\right)^2\right]$

(b) $\displaystyle\lim_{x\to\infty}\frac{1+x+\cos x}{(x+\cos x)e^{\sin x}}$

(c) $\displaystyle\lim_{x\to\infty}x\left(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x\right)$

(19) [15pt] Evaluate

(a) $\displaystyle\int\frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx$ (b) $\displaystyle\int_0^{\pi/2}\frac{1}{(3\cos x+4\sin x)^2}dx$ (c) $\displaystyle\int_2^{\infty}\frac{7x^2+1}{(x^2-1)(x^2+1)^2}dx.$

(20) [7pt] Let $(x(t),y(t))$ parametrize the function curve of $y=f(x)$ such that $x(0)=0$, $y(0)=1$. Suppose $x(t),y(t)$ satisfy the following conditions:

$$\begin{cases}\displaystyle\int_0^x\frac{1}{\sqrt{4-u^3}}\,du &= t\\ \ln y+3-3y &= t\end{cases}$$

Find the tangent line of $y=f(x)$ at $x=0$. What is the concavity of $f(x)$ at $x=0$?

(21) [7pt] Find the relative extreme values and the absolute extreme values of

$$f(x)=\left(1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}\right)e^{-x},\quad x\in\mathbb{R}$$

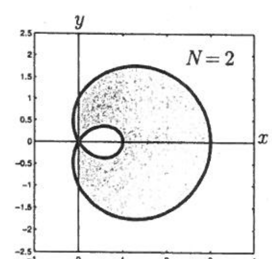
where n is a positive integer.

(22) [7pt] Find approximations of all solutions of the equation $x^5+5x+7=0$ with error ≤ 0.05. You should verify your estimation.

(23) [8pt] Consider the polar curve $r=1+N\cos\theta$, N is a natural number (the figure at right is for the case $N=2$). Let A_N be the area of the shaded region.

(a) Compute A_N.

(b) Find the limit $\displaystyle\lim_{N\to\infty}\frac{A_N}{N}$.



(24) [8pt] Show that if f is a continuous function on $[0,1]$ such that $\displaystyle\int_0^1 f(x)^2dx=0$, then $f(x)=0$ for all $x\in[0,1]$.

(25) [8pt] Let $f(x)$ be a continuous function on $(0,\infty)$ which is not identical to zero. Suppose that $f(xy)=f(x)+f(y)$, for all $x,y>0$. Show that $f(x)=\log_a x$ for some positive number a.

(26) [9pt] Sketch the graph of the function $f(x)=\ln\left|\dfrac{x+1}{x-1}\right|$. (You must discuss the following: asymptotes; intervals of increase or decrease; local maximum and minimum values; intervals of concavity and the inflection points).