

微積分甲認證考試內容大綱(英文版)

Prerequisite 1: Quadratic curves (standard form).

Prerequisite 2: Vectors (inner product, cross product, line and plane in vector form).

PART 1

1. Functions

1.1 Definition, polynomial functions, rational functions, trigonometric functions, exponential functions, logarithmic functions.

1.2 Operations and compositions of functions.

1.3 Inverse functions, inverse trigonometric functions.

2. Limits

2.1 Limit of functions.

2.2 Laws of limits, squeeze theorem.

2.3 Continuous functions, limits of composite functions.

2.4 The precise definition of a limit in terms of ε δ .

2.5 Intermediate value theorem for continuous functions.

3. Derivatives

3.1 Derivatives, rates of change, tangent line.

3.2 Differentiation rules, chain rule.

3.3 Derivatives of elementary functions.

3.4 Higher derivatives.

3.5 Implicit differentiation, derivatives of inverse functions, logarithmic differentiation.

4. Applications of differentiation

4.1 Rolle's theorem, mean value theorem.

4.2 Monotonicity, concavity, extrema, inflection points and asymptotes of functions.

4.3 First order test and second order test for extrema.

4.4 Graphing functions.

4.5 Optimization.

4.6 L'Hospital's rule.

4.7 Related rates.

4.8 Linear approximation and differentials.

4.9 Newton's method of finding roots.

5. Integrations

5.1 Riemann sums and definite integrals.

5.2 Fundamental theorem of calculus.

5.3 Indefinite integrals (anti-derivatives)

- 5.4 Integration by substitutions.
- 5.5 Integration by parts.
- 5.6 Integration of rational functions.
- 5.7 Trigonometric substitutions.
- 5.8 Techniques of integration.

6. Applications of integration

- 6.1 Areas between curves.
- 6.2 Volume (disc method and shell method).
- 6.3 Arc length and the surface area of revolution.
- 6.4 The average value of a function.
- 6.5 Center of mass, centroid, Pappus theorem.
- 6.6 Works in physics.
- 6.7 Improper integral.
- 6.8 Numerical integration: rectangular method, trapezoidal rule, simpson rule (including error estimation)
- 6.9 First order linear differential equations.
- 6.10 Separable equations.

7. Parametric equations and polar coordinate

- 7.1 Parametric curves in the x - y plane.
- 7.2 The calculus in parametric forms: tangent line, area, arc length, the area of a surface generated by revolution.
- 7.3 Polar coordinate.
- 7.4 Tangent lines, length and area in polar coordinate.
- 7.5 Quadratic curves in polar coordinate.

PART 2

8. Infinite series

- 8.1 Infinite sequences and infinite series.
- 8.2 Integral test, comparison test.
- 8.3 Alternating series.
- 8.4 Absolute convergence, ratio test, root test.
- 8.5 Power series, radius of convergence, operations on power series.
- 8.6 Taylor series and applications.
- 8.7 Approximation of functions by Taylor polynomials.

9. Vector valued functions

- 9.1 Vector valued functions.
- 9.2 Differentiation rules of vector valued functions.
- 9.3 Curves: tangent vector, arc length, curvature.
- 9.4 Motion in space: position, velocity and acceleration.

9.5 Kepler's laws of planets.

10. Partial derivatives

10.1 Function of several variables.

10.2 Graph of function of several variables, level curves and level surfaces.

10.3 Limit and continuity.

10.4 Partial derivatives.

10.5 Higher derivatives.

10.6 Tangent planes.

10.7 Chain rule.

10.8 Directional derivatives and gradient.

10.9 Extrema and saddle points, the second order test of extrema.

10.10 Conditioned extrema, Lagrange multipliers.

10.11 Linear approximations and differentials.

11. Multiple integrals

11.1 Double integral, iterated integral and Fubini theorem.

11.2 Double integral in polar coordinate.

11.3 Center of mass, moment of inertia.

11.4 Triple integrals and Fubini theorem.

11.5 Cylindrical coordinate and spherical coordinate of space.

11.6 Triple integral in cylindrical coordinate.

11.7 Triple integral in spherical coordinate.

11.8 Change of variables of multiple integrals, Jacobian.

12. Vector calculus

12.1 Vector fields.

12.2 Line integral, fundamental theorem of line integrals.

12.3 Green theorem.

12.4 Curl (curl) and divergence (div).

12.5 Surfaces in parametric form, surface area and surface integrals.

12.6 Stokes' theorem.

12.7 Divergence theorem.